

**B.Tech. Project Report (Final Stage)**

# **Prediction of CMC and Surface Tension in Mixed Surfactant Solutions**

By

**Mahesh Kumar Inakhiya**

**Roll No : 98002033**

Under the guidance of

**Prof. Sandip Roy**

Department of Chemical Engineering

Indian Institute of Technology

Mumbai

April 19, 2002

# Acknowledgement

I take this opportunity to express my sincere gratitude to my guide **Prof. Sandip Roy** for guiding me in the best possible way throughout the course of this project and without whose help this report would never have been possible.

April 19, 2002

Mahesh Kumar Inakhiya

### **Abstract**

Surfactant mixtures are commonly used in many practical surfactant applications. Surfactant Mixtures are often advantageous because they show the synergism in the system. Our B. Tech project deals with the prediction of the CMC and Surface tension in mixed surfactant system. We have reviewed total 4 model for the CMC and Surface tension prediction. We have predicted the values of CMC and Surface tension by 2 models. Predicted values are in good agreement with the values reported in the literature.

# Contents

<b>Acceptance Certificate</b>	<b>i</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Basic definitions . . . . .	1
<b>2 Prediction Of CMC</b>	<b>3</b>
2.1 Pseudophase separation model-Thermodynamic Framework . . .	3
2.1.1 Gibbs free energy and chemical potentials . . . . .	3
2.2 Composition Distribution of mixed micelles . . . . .	6
2.2.1 Molecular model for $g_{mic}$ calculation . . . . .	8
2.2.2 CMC prediction . . . . .	12
2.3 Regular solution approximation model . . . . .	13
2.3.1 Ideal Treatment . . . . .	13
2.3.2 General Treatment . . . . .	14
<b>3 Prediction of Surface tension</b>	<b>17</b>
3.1 Rigorous Model . . . . .	17
3.2 Simple Model . . . . .	21
3.2.1 Non Ideal Mixture of homologues . . . . .	21
3.2.2 Mixtures of ionic homologues . . . . .	22
3.2.3 Mixture of components with different molar areas . . . . .	23

3.3	Other Model . . . . .	24
<b>4</b>	<b>Results</b>	<b>26</b>
4.1	CMC prediction . . . . .	26
4.2	Surface Tension Prediction . . . . .	27
<b>5</b>	<b>Conclusions</b>	<b>28</b>

# Acceptance Certificate

Department of Chemical Engineering  
Indian Institute of Technology, Bombay

This B.Tech Project Final stage report titled “ Prediction Of CMC and Surface Tension in the Mixed Surfactant Solutions.” by Mahesh Kumar Inakhiya (Roll Number: 98002033) may be accepted.

April 2002

Prof. Sandip Roy

# List of Figures

2.1	Schematic representation of thought process to visualize the various physicochemical factors involved in the formation of a mixed micelle. Surfactant A is represented by a black head and a full tail and surfactant B by a blank head and dashed tail. . . . .	9
-----	--	---

# List of Tables

# Chapter 1

## Introduction

Surfactant mixtures are commonly used in many practical surfactant applications. Mixtures are advantageous because purification of a single component may be too costly or difficult and surfactant mixtures often perform better than a single surfactant. The synergistic behaviour of a surfactant mixtures may also be exploited to reduce the total amount of surfactant used in a particular application, thus reducing both cost and environmental impact. In addition, as environmental impact regulations on producing and releasing new materials become more restrictive, it may be preferable from a regulatory perspective to combine existing surfactants rather than to introduce new ones.

For that surfactant mixtures solution properties should be known, and for some mixtures experimental data are not available.

Our project dealt with the various models available in the literature for the prediction of critical micelle concentration and surface tension in mixed micelle system.

### 1.1 Basic definitions

Micelle: Micelle is the colloidal aggregate of amphipathic (surfactant) molecules, which occurs at a well-defined concentration known as the critical micelle concentration. The typical number of aggregated molecules in a micelle (aggregation number) is 50 to 100.

Surfactant: surfactant is a soluble compound that reduces the surface tension of liquids, or reduces interfacial tension between two liquids or a liquid and a solid. It is also known as a surface active agent. The term is derived from SURFace ACTive AgeNT. It is a compound that contains a Hydrophilic and a Hydrophobic segment.

HLB: HLB is an arbitrary scale from 0 to 40 depicting the Hydrophilic/Lipophilic Balance of a surfactant. Products with low HLB are more oil soluble. High HLB represents good water solubility. HLB is a numerically calculated number based on the surfactants molecular structure. It is not a measured parameter.

Moles of Ethylene Oxide: The Ethylene Oxide (EO) chain is the hydrophilic portion of the Surfactant molecule. The larger this portion of the molecule, the more water soluble is the non-ionic surfactant. Ethylene oxide is the reactive chemical added to base alcohols and amines to form ethoxylated non-ionic surfactants. Base Alcohol (Hydrophobic) Ethylene Oxide chain (Hydrophilic)

This surfactant contains  $n$  moles of Ethylene Oxide. Where surfactants are used to emulsify oils the Hydrophobic part of the surfactant embeds itself in the oil droplet and the Hydrophilic Ethylene oxide chain interacts with water to surround the oil droplet and form an emulsion. Surfactant are classified as follows

Nonionic Anionic

Mixed micelles: When there is more than one surfactant in the aqueous solution than the mixed micelles are formed.

# Chapter 2

## Prediction Of CMC

### 2.1 Pseudophase separation model-Thermodynamic Framework

#### 2.1.1 Gibbs free energy and chemical potentials

The thermodynamic formulation used to describe the free energy of a mixed surfactant solution constitute a generalization of the one developed to describe the single surfactant solutions.<sup>[9]</sup> This formulation is for the aqueous solution of two surfactants. Considering a solution of  $N_W$  water molecules,  $N_A$  surfactant A molecules, and  $N_B$  surfactant B molecules in the thermodynamic equilibrium at temperature T and pressure P. If the concentration of the surfactant mixture exceeds its CMC, the surfactant molecules will assemble to form a distribution of mixed micelles  $N_{n\alpha}$  having aggregation number n and composition  $\alpha$ . In such a mixed micelle there are  $n\alpha$  surfactant A molecules and  $n(1 - \alpha)$  surfactant B molecules. and

$$N_A = \sum_{n,\alpha} n\alpha N_{n\alpha} \quad (2.1)$$

$$N_B = \sum_{n,\alpha} n(1 - \alpha) N_{n\alpha} \quad (2.2)$$

Mixed micelles of different size and composition are treated as distinct species in chemical equilibrium with each other as well as with the free monomers in the solution.

The gibbs free energy of the mixed surfactant solution  $G$  is modeled as the sum of three contributions: the free energy of formation  $G_f$ , the free energy of mixing  $G_m$ , and the free energy of interaction  $G_i$ .

The free energy of formation is expressed as

$$G_f = N_w \mu_w^o + N_A \mu_A^o + N_B \mu_B^o + \sum_{n,\alpha} n N_{n\alpha} g_{mic}(sh, n, \alpha) \quad (2.3)$$

where  $\mu_w^o(T, P)$ ,  $\mu_A^o(T, P)$ , and  $\mu_B^o(T, P)$  are the standard state chemical potentials of water, surfactant A monomers and Surfactant B monomers respectively, at the solution temperature  $T$  and pressure  $P$ ;  $g_{mic}(sh, n, \alpha)$  is the free energy of mixed micellization, which represents the free energy change per monomer associated with transferring  $n\alpha$  surfactant A monomers and  $n(1 - \alpha)$  surfactant B monomers from water into a mixed micelles of size  $sh$ , aggeration number  $n$ , and composition  $\alpha$ .

The free energy of mixing the formed mixed micelles, free monomers, and water is modeled by an expression of the form

$$G_m = kT [N_w \ln X_w + \sum_{n,\alpha} N_{n\alpha} \ln X_{n\alpha}] \quad (2.4)$$

where  $X_w = N_w / (N_w + N_B + N_A)$ ,  $X_{n\alpha} = N_{n\alpha} / (N_w + N_A + N_B)$   $k$  is the Boltzmann constant, and  $T$  is the absolute temperature.  $-G_m/T$  is an entropic contribution which reflects the number of ways in which the distribution of mixed micelles, the free monomers, and water molecules can be positioned in the solution as a function of the solution concentration and composition.

The free energy of interaction reflects interaction between mixed micelles, water molecules, and free monomers in the solution. Free energy of interaction is in the following form

$$G_i = -1/2 C_{eff}(\alpha_{soln}) (N_A + N_B) \phi \quad (2.5)$$

where  $\phi = \phi_A + \phi_B$  is the sum of the volume fractions,  $\phi_A$  and  $\phi_B$  of surfactants A and B, respectively  $\alpha_{soln} = N_A / (N_A + N_B)$  is the composition of the surfactant mixture, and  $C_{eff}(\alpha_{soln})$  is an effective mean-field interaction parameter for

the mixture which is related to the single surfactant interaction parameter  $C_{AB}$  through the following expression

$$C_{eff}(\alpha_{soln}) = C_{AW}\alpha_{soln} + C_{BW}(1 - \alpha_{soln}) - C_{AB}\alpha_{soln}(1 - \alpha_{soln})(\sqrt{\gamma_A\gamma_B}/\gamma_{eff}) \quad (2.6)$$

in equation 1.6,  $\gamma_A = \Omega_A/\Omega_w$  and  $\gamma_B = \Omega_B/\Omega_w$ , where  $\Omega_w$ ,  $\Omega_A$ , and  $\Omega_B$  are the effective molecular volumes of water, surfactant A, and surfactant B, respectively, and  $\gamma_{eff} = \alpha_{soln}\gamma_A + (1 - \alpha_{soln})\gamma_B$ . The CMC of mixed micelles can be predicted by the above free energy model through the chemical potential of water,  $\mu_w$ , and the chemical potential of a mixed micelle, of aggregation number  $n$  and composition  $\alpha$ ,  $\mu_{n\alpha}$ , which are obtained by differentiating the gibbs free energy, eqs 1.3-1.5, with respect to  $N_w$  and  $N_{n\alpha}$ , respectively.<sup>[9]</sup> The resulting expression are given by

$$\mu_w = \left( \frac{\partial G}{\partial N_w} \right)_{T,P,N_{n\alpha}} = \mu_w^o + kT[\ln(1 - X) + X - \sum_{n,\alpha} X_{n\alpha}] + C_{eff}(\alpha_{soln}) \left( \frac{\phi^2}{2\gamma_{eff}} \right) \quad (2.7)$$

$$\mu_{n\alpha} = \left( \frac{\partial G}{\partial N_{n\alpha}} \right)_{T,P,N_w,N_{n\alpha}} = n\alpha\mu_A^o + n(1 - \alpha)\mu_B^o + (ng_{mic} + kT) + kT(\ln X_{n\alpha} + n(X - 1 - \sum_{n\alpha} X_{n\alpha})) + n\alpha\mu_A^i + n(1 - \alpha)\mu_B^i \quad (2.8)$$

where  $X = X_A + X_B$ , with  $X_A = N_A/(N_w + N_A + N_B)$  and  $X_B = N_B/(N_w + N_A + N_B)$ , is the total mole fraction of surfactant in the solution, and the interaction contributions to the monomer chemical potentials are given by

$$\mu_A^i = \frac{\phi}{2} \left[ C_{Aw} + \frac{\gamma_A}{\gamma_{eff}} [\alpha_{soln}C_{Aw} + (1 - \alpha_{soln})C_{Bw}] (1 - \phi) - (1 - \alpha_{soln}) \frac{\sqrt{\gamma_A\gamma_B}}{\gamma_{eff}} C_{AB} (1 - \phi_A) \right] \quad (2.9)$$

$$\mu_B^i = \frac{\phi}{2} \left[ C_{Bw} + \frac{\gamma_B}{\gamma_{eff}} [\alpha_{soln}C_{Aw} + (1 - \alpha_{soln})C_{Bw}] (1 - \phi) - \alpha_{soln} \frac{\sqrt{\gamma_A\gamma_B}}{\gamma_{eff}} C_{AB} (1 - \phi_B) \right] \quad (2.10)$$

The chemical potentials of the surfactant A and surfactant B monomers is obtained from eq 1.8 by substituting  $n = 1$  and  $\alpha = 1$  (for A) or 0 (for B), respec-

tively, that is

$$\mu_A = (\mu_A^o + kT) + kT[\ln X_{1A} + X - 1 - \sum_{n\alpha} X_{n\alpha}] + \mu_A^i \quad (2.11)$$

$$\mu_B = (\mu_B^o + kT) + kT[\ln X_{1B} + X - 1 - \sum_{n\alpha} X_{n\alpha}] + \mu_B^i \quad (2.12)$$

where  $X_{1A}$  and  $X_{1B}$  are the mole fractions of free surfactant A and surfactant B monomers, respectively. Using eqs. 1.7-1.12 for the chemical potentials, one predicts the CMC of the mixed micellar solution in the following way.

## 2.2 Composition Distribution of mixed micelles

When the mixed micellar solution is in the thermodynamic equilibrium, the chemical potential  $\mu_{n\alpha}$  of a mixed micelle of aggregation number  $n$  and composition  $\alpha$  is related to the chemical potentials of the free monomers through the constraints imposed by the conditions of multiple chemical equilibrium, that is  $\mu_{n\alpha} = n\alpha\mu_A + n(1-\alpha)\mu_B$  (2.12) equation 2.13 implies that the chemical potential of a mixed micelle having aggregation number  $n$  and composition  $\alpha$  is equal to the sum of the chemical potentials of its constituent  $n\alpha$  surfactant A and  $n(1-\alpha)$  surfactant B molecules. Substituting eqs 2.8, 2.11 and 2.12 in eq 2.13, one obtains the following expression for the equilibrium micellar size and composition distribution

$$X_{n\alpha} = \frac{1}{e} X_{1A}^{n\alpha} X_{1B}^{n(1-\alpha)} e^{-n[\beta g_{mic}(\alpha) - 1]} = \frac{1}{e} X_1^n e^{-n\beta g_m(\alpha, \alpha_1)} \quad (2.13)$$

where  $\beta = \frac{1}{kT}$ ,  $\beta g_m = [\beta g_{mic} - 1 - \alpha \ln \alpha_1 - (1-\alpha) \ln (1-\alpha_1)]$  is a modified dimensionless free energy of the mixed micellization per monomer,  $X_1 = X_{1A} + X_{1B}$  is the total mole fraction of free monomers in solution.

The composition  $\alpha^*(n)$ , at which  $X_{n\alpha}$  exhibits a maximum for a given micellar aggregation number  $n$ , is referred as the optimum composition.  $\alpha^*(n)$  is function of the aggregation number  $n$  and one can obtain it by setting the derivative of  $X_{n\alpha}$  with respect to  $\alpha$  equal to zero. By implementing this procedure one can obtain the following implicit equation

$$\beta \frac{\partial g_{mic}(n, \alpha)}{\partial \alpha} \Big|_{\alpha^*} = \ln \frac{\alpha_1}{(1-\alpha_1)} \quad (2.14)$$

using eq 2.14 one can obtain the composition for all aggregation numbers from the knowledge of  $\alpha_1$  and  $g_{mic}(n, \alpha)$ .

Equation 2.13 for the micellar size and composition distribution is applicable to mixed micelles of all shapes, sizes, and compositions. However, as shown in equation 2.13 one needs to know

(i) the free energy of micellization  $g_{mic}(n, \alpha)$  as a function of  $n$  and  $\alpha$  or, equivalently,  $g_m$  as a function of  $n$ ,  $\alpha$ , and  $\alpha_1$ ,

(ii) the equilibrium solution monomer mole fraction  $X_1$ , and

(iii) the equilibrium solution monomer composition  $\alpha_1$ .

condition (ii) and (iii) are equivalent to knowing the  $X_{1A} = \alpha_1 X_1$  and  $X_{1B} = (1 - \alpha_1) X_1$ .

$X_1$  and  $\alpha_1$  (or equivalently  $X_{1A}$  and  $X_{1B}$ ) is calculated by using the eq. 2.13 in two constraint imposed by the conservation of the total number of surfactant A and surfactant B molecules in solution, that is,  $N_A = \sum_{n\alpha} n\alpha N_{n\alpha}$  and  $N_B = \sum_{n\alpha} n(1 - \alpha) N_{n\alpha}$ , or equivalently

$$X_A = \alpha_{soln} X = \alpha_1 X_1 + \sum_{n,\alpha} n\alpha X_{n\alpha} \quad (2.15)$$

$$X_B = (1 - \alpha_{soln}) X = (1 - \alpha_1) X_1 + \sum_{n,\alpha} n(1 - \alpha) X_{n\alpha} \quad (2.16)$$

given  $g_{mic}$  (or equivalently  $g_m$ ), and on inserting eq. 2.13 into eqs 2.15 and 2.16, one obtains two implicit equations for  $X_1$  and  $\alpha_1$  as a function of  $X$  and  $\alpha_{soln}$ . Solving these two equations simultaneously one can obtain  $X_1(X, \alpha_{soln}, T, P)$  and  $\alpha_1(X, \alpha_{soln}, T, P)$  which is inserted back into the eq 2.13 to calculate the entire micellar size and composition distribution  $X_{n\alpha}$  as a function of  $X, \alpha_{soln}, T, P$ , and other solution conditions.

$g_{mic}$  is calculated by following molecular model

### 2.2.1 Molecular model for $g_{mic}$ calculation

The free energy of mixed micellization  $g_{mic}(sh, n, \alpha, l_c)$  represents the free energy change (per monomer) associated with creating a micelle, having shape  $sh$ , aggregation number  $n$ , composition  $\alpha$ , and core minor radius  $l_c$ , from  $n\alpha$  A-type and  $n(1 - \alpha)$  B-type surfactant monomers.

The magnitude of  $g_{mic}$  reflects many complex physicochemical factors such as the hydrophobic effect, interfacial effects, conformational free energy changes associated with restricting the hydrophobic chains inside the micellar core, steric and electrostatic interactions between the hydrophilic moieties at the micellar core-water interface, and entropy effects associated with mixing the two surfactant species in a mixed micelle.<sup>[10]</sup>

For the evaluation of free energy contribution associated with the various physicochemical factors mentioned above, one can visualize the reversible formation of mixed micelle having shape  $sh$ , aggregation number  $n$ , composition  $\alpha$ , and core minor radius  $l_c$  (final state) from  $n\alpha$  surfactant A monomers and  $n(1 - \alpha)$  surfactant B monomers (initial state) in water, as shown in Figure 2.1. The various steps of the mixed micelles formation process is as follows:

In the first step the heads, if charged are discharged along with the counterions. Subsequently, in the second step, the bond between the head and the tail of each surfactant molecule is broken, in the third step, the hydrocarbon tails of surfactant A and B are transferred from water to a mixture of hydrocarbons A and B whose composition is equal to the micellar composition  $\alpha$ . In the fourth step, an hydrocarbon droplet having shape  $sh$  and core minor radius  $l_c$  is created from the hydrocarbon mixture having composition  $\alpha$ . That is, in this step, an interface separating the hydrocarbon mixture from water is created. Within this hydrocarbon droplet the tails are unrestricted and can move freely. However in a micelle, each tail is bonded to a head and therefore one of the tail ends is restricted to lie in the vicinity of the micellar core-water interface. Accordingly, in the fifth step, this restriction is imposed on the tails. So, at the end of the fifth step, the creation of micellar core has been completed. The creation of micellar corona of heads follows next. Accordingly, in the sixth step, the discharged heads are reattached to the tails at the micellar core-water interface. This involves three operations:

Figure 2.1: Schematic representation of thought process to visualize the various physicochemical factors involved in the formation of a mixed micelle. Surfactant A is represented by a black head and a full tail and surfactant B by a blank head and dashed tail.

recreating the bond between the head and tail, screening part of the micellar core-water interface from contact with water, and introducing steric repulsion between the heads. Finally, in the seventh step, the heads if charged, are recharged along with the associated counterions. This completes the creation of the micellar corona and hence of the entire mixed micelle.<sup>[10]</sup>

Various contributions to the  $g_{mic}$  is related to the various physicochemical factors associated with the micellization.

These include:

- (i) The hydrophobic free energy  $g_{w/hc}$  associated with transferring the hydrocarbon tails from water to an hydrocarbon mixture in the third step,
- (ii) The interfacial free energy  $g_{\sigma}$  associated with creating the micellar core-water interface in the fourth step, as well as with shielding part of that interface in the sixth step,
- (iii) The configurational (packing) free energy  $g_{hc/mic}$  arising from the loss in configurational degrees of freedom in the fifth step,
- (iv) The steric free energy  $g_{st}$  associated with repulsive steric interactions between the heads in the sixth step, and
- (v) the electrostatic free energy  $g_{elec}$  associated with the first and the seventh steps.

The total free energy of mixed micellization  $g_{mic}(sh, n, \alpha, l_c)$  is then computed by summing these five free energy contributions, that is

$$g_{mic}(sh, n, \alpha, l_c) = g_{w/hc} + g_{\sigma} + g_{hc/mic} + g_{st} + g_{elec} \quad (2.17)$$

using Eq 2.17 one can calculate the free energy of micellization for the three regular micellar shapes of spheres, infinite-sized cylinders, and infinite-sized disks or bilayers.

Five energy contributions in the above equation is calculated in the following way:

#### A. Hydrophobic Free Energy

$g_{w/hc}$  represents the free energy change associated with transferring the hydrocarbon tails of surfactants A and B from water to a mixture of hydrocarbons A and B whose composition is equal to micellar composition  $\alpha$ .<sup>[10]</sup>

For calculations

$$g_{w/hc}(\alpha) = \alpha g_{w/hc}^A + (1 - \alpha) g_{w/hc}^B + kT[\alpha \ln \alpha + (1 - \alpha) \ln (1 - \alpha)] \quad (2.18)$$

where

$$g_{w/hc}^A = [(4.09 - 1.05n_{cA})(298/T) - (4.62 + 0.44n_{cA})]kT \quad (2.19)$$

and

$$g_{w/hc}^B = [(4.09 - 1.05n_{cB})(298/T) - (4.62 + 0.44n_{cB})]kT \quad (2.20)$$

### B. Interfacial Free Energy

The interfacial free energy  $g_\sigma$  reflects the contribution to  $g_{mic}$  associated with the creating a micellar core-water interface.<sup>[10]</sup>

For calculations

$$g_\sigma = \alpha\sigma_A(a - a_{0A}) + (1 - \alpha)\sigma_B(a - a_{0B}) \quad (2.21)$$

where  $\sigma_A$  and  $\sigma_B$  are the curvature-dependent interfacial tensions between hydrocarbons A and B, respectively,  $a_{0A}$  and  $a_{0B}$  are the corresponding interfacial areas per surfactant molecule screened by the heads (approximately equal to  $21 A^2$  each for single tail surfactants), and  $a = Sv/l_c$  is the available interfacial area per surfactant molecule, where  $S$  is shape factor (3 for spheres, 2 for cylinders, and 1 for disks or bilayers),  $v = \alpha v_A + (1 - \alpha)v_B$  is an average tail volume, where

$$v_A = 27.4 + 26.9(n_{cA} - 1) \quad (2.22)$$

and

$$v_B = 27.4 + 26.9(n_{cB} - 1) \quad (2.23)$$

$\sigma_A$  and  $\sigma_B$  are calculated from the following eqs.

$$\sigma_A = \sigma_{0A}[1 - (S - 1)\delta_A/l_c] \quad (2.24)$$

and

$$\sigma_B = \sigma_{0B}[1 - (S - 1)\delta_B/l_c] \quad (2.25)$$

where  $\sigma_{0A}$  and  $\sigma_{0B}$  are the interfacial tensions at a planar interface between water and hydrocarbons A and B.  $\delta_A$  and  $\delta_B$  are the Tolman distance corresponding to hydrocarbons A and B. It is calculated as follows. for  $C_{11}$ ,  $\delta = 2.25A^o$  and for  $C_n$ ,  $\delta(n) = \delta(11)l_{max}(n)/l_{max}(11)$ , where  $l_{max}(n) = 1.54 + 1.265n$ ,  $n$  is number of carbon atoms.

### C. Packing Free Energy

In a mixed micelle, the nonpolar hydrocarbons tails bonded to the heads, and therefore the tail ends which are attached to the heads are restricted to lie in the vicinity of the micellar core-water interface. This results in a loss of conformational degree of freedom, and the associated free energy change  $g_{hc/mic}$  is evaluated using a single chain mean field model.<sup>[10]</sup>

For calculation purpose,

$$-\frac{g_{hc/mic}}{kT} = (Z_A/Z^{free}_A) + (1 - \alpha)\ln(Z_B/Z^{free}_B) + \rho_{hc} \sum_{i=1}^L F_i V_i \quad (2.26)$$

where  $Z$  is the partition function associated with the single chain of type A or B,  $F_i$  is the mean field,  $\rho_{hc}$  is density of  $CH_2$  groups in the micellar core.

### D. Steric Free Energy

The steric free energy contribution is calculated by treating the heads present at interface as a localized monolayer, which reflects the fact that each head is physically attached to a tail at the interface.<sup>[10]</sup>

For calculations

$$g_{st} = -kT \ln \left[ 1 - \frac{\alpha a_{hA} + (1 - \alpha) a_{hB}}{a} \right] \quad (2.27)$$

where  $a_{hA}$  and  $a_{hB}$  are the average cross sectional area of head A and B, respectively.

### E. Electrostatic Free Energy

Electrostatic free energy contribution is often very small. So it is generally not included in the  $g_{mic}$  calculations.

## 2.2.2 CMC prediction

At very low surfactant concentration most of the surfactant molecules exist as free monomers. However, as the total concentration of surfactant is increased, keeping its composition constant, micelles begin to form beyond a certain threshold concentration known as Critical micellar concentration (CMC). Beyond the CMC most of the added surfactant remains in the micellar form, and the total

monomer concentration remains practically constant. The first micelle that form has a composition close to the optimum value  $\alpha^*$ , because at  $\alpha = \alpha^*$  the free energy of micellization exhibits a maximum. The mole fraction of these micelles can therefore be expressed, using  $\alpha = \alpha^*$  in eq. 12.3 as

$$X_{n\alpha^*} = \frac{1}{e} \left( \frac{X_1}{e^{\beta g_m(\alpha^*, \alpha_1)}} \right)^n \quad (2.28)$$

Using the expression for the  $g_m$  one finds that

$$\ln(\text{cmc}) \approx \beta g_m(\text{sh}, \alpha^*, \alpha_1) = \beta g_{mic}^B + \beta g_{mic}^{AB} \alpha^{*2} + \ln \left( \frac{1 - \alpha^*}{1 - \alpha_1} \right) - 1 \quad (2.29)$$

where sh corresponds to the shape of the optimum micelle, and  $\alpha^*$  is obtained from the following equation

$$\beta(g_{mic}^A - g_{mic}^B) + \beta g_{mic}^{AB}(1 - 2\alpha^*) + \ln \frac{\alpha^*}{1 - \alpha^*} = \ln \frac{\alpha_1}{1 - \alpha_1} \quad (2.30)$$

By combining the Eqs. 2.18 and 2.19 one obtains the

$$\frac{1}{\text{cmc}} = \frac{\alpha_1}{f_A \text{cmc}_A} + \frac{1 - \alpha_1}{f_B \text{cmc}_B} \quad (2.31)$$

where

$$\ln f_A = \beta g_{mic}^{AB}(1 - \alpha^*)^2,$$

$$\ln f_B = \beta g_{mic}^{AB}(\alpha^*)^2,$$

$$\ln(\text{cmc}_A) = \beta g_{mic}^A - 1,$$

$$\text{and } \ln(\text{cmc}_B) = \beta g_{mic}^B - 1.$$

so one can predict the mixtures CMC from a knowledge of  $g_{mic}^A$ ,  $g_{mic}^B$ , and  $g_{mic}^{AB}$ .

## 2.3 Regular solution approximation model

### 2.3.1 Ideal Treatment

When two surfactant with the same head group but with different chain lengths are mixed, then there is no net interaction between the surfactant species. There

is interaction between the head groups of the surfactants in the mixed micelles, but since the head groups are of the same kind this interaction is not different for the different surfactant species so because of this the net interaction is zero.<sup>[5]</sup>

CMC of a surfactant is an average of the CMC's of the single surfactants,

$$CMC = x_1 CMC_1 + (1 - x_1) CMC_2 \quad (2.32)$$

where CMC is the critical micelle concentration of the surfactant mixture and the  $CMC_i$  are the critical micelle concentration of the single surfactant species and  $x_1$  is the fraction of the surfactant 1 in the whole system, *i.e.* where

$$x_1 = \frac{C_1}{C_1 + C_2} \quad (2.33)$$

$C_1$  and  $C_2$  are the molar surfactant concentrations of respective species.

If  $x_1^m$  represents the mole fraction of surfactant 1 in micelle itself and not in the whole system, then the CMC of a surfactant mixture is

$$CMC = x_1^m CMC_1 + (1 - x_1^m) CMC_2 \quad (2.34)$$

where  $x_1^m$  is the mole fraction of surfactant 1 in micelles. the molar composition  $x_1^m$  of mixed micelle is given by

$$x_1^m = \frac{x_1 CMC_2}{x_1 CMC_2 + (1 - x_1) CMC_2} \quad (2.35)$$

### 2.3.2 Genaral Treatment

In many systems surfactants of different kinds are mixed, such as anionic and nonionics. Here the nonionic surfactants shield the repulsion between the head groups of the anionic surfactants in the micelle and hence there is a net interaction between the two type of surfactant. Another example is of mixtures of anionic and cationic surfactants, where exists a very srtrong interacton between the surfactants. For these type of mixtures the CMC is given by following equation

$$CMC = x_1^m f_1 CMC_1 + (1 - x_1^m) f_2 CMC_2 \quad (2.36)$$

where  $f_1$  and  $f_2$  are the activity coefficients of the surfactants in the micelle. For calculation of these activity coefficients following equation are used

$$\ln f_1 = (1 - x_1^m)^2 \beta \quad (2.37)$$

$$\ln f_2 = (x_1^m)^2 \beta \quad (2.38)$$

where  $\beta$  is an interaction parameter, quantifying the net interaction between the surfactant species in the micelle. Positive  $\beta$  value imply that there is a net repulsion between two surfactant components while negative  $\beta$  value imply a net attraction. If  $\beta$  is zero the activity coefficient will be unity and equation 2.25 will revert to equation 2.23. Negative  $\beta$  values are most commonly found, significant for a net attraction between the surfactant species. Positive  $\beta$  values also exists *e.g.* mixtures of normal hydrocarbon-based surfactants with fluorinated ones.

in terms of solution compositions

$$\frac{1}{CMC} = \frac{x_1}{f_1 CMC_1} + \frac{1 - x_1}{f_2 CMC_2} \quad (2.39)$$

and

$$x_1^m = \frac{x_1 f_1 CMC_2}{x_1 f_2 CMC_2 + (1 - x_1) f_1 CMC_1} \quad (2.40)$$

For various surfactant mixtures  $\beta$  values are given in Table 2.1.<sup>[7]</sup>

$\beta$  values

# Chapter 3

## Prediction of Surface tension

The ability of surfactants to adsorb at liquid interfaces is an important property as surfactant compositions are widely used for many technological processes, such as detergents, foam and emulsion stabilizers, etc. These compositions are usually mixtures of homologues or surfactants of different nature, e.g. non-ionic surfactants with ionics, ionic surfactants with zwitterionic ones, or anionic and cationic surfactants. All models proposed so far for the prediction of mixture properties dealt with the predictions of surface or interfacial tensions of mixed solutions from known characteristics of the individual compounds. Therefore, in all equations of state proposed for mixed surface layers, the isotherm parameters of pure compounds were involved.

This model predicts the surface tension of a surfactant mixture from the surface tension values or isotherm parameters of individual solution by the use of one simple equation. This equation can be applied not only to ideal mixtures of homologues, but also to surfactants which exhibit a non-ideal surface behaviour.

### 3.1 Rigorous Model

Equations of state for surface layers, adsorption isotherms and surface tension isotherms can be derived by equating the expressions for the chemical potentials at the surface  $\mu_i^s$ ,

$$\mu_i^s = \mu_i^{os} + RT \ln f_i^s x_i^s - \gamma \omega_i \quad (3.1)$$

to those in the solution bulk

$$\mu_i^\alpha = \mu_i^{o\alpha} + RT \ln f_i^\alpha x_i^\alpha \quad (3.2)$$

where R is the gas law constant, T is the temperature,  $\gamma$  is surface tension,  $f_i$  are the activity coefficients,  $x_i$  are the molar fractions,  $\omega_i$  are the partial molar surface areas. The superscript ' $\alpha$ ' refers to the bulk solution, and 's' to the surface; the standard chemical potentials  $\mu_i^{o\alpha}$  and  $\mu_i^{os}$  depend on pressure and temperature. This method yields the equation of state and adsorption isotherm for mixture of two non-ionic surfactants (i=1,2) in non-ideal surface layers:<sup>[1]</sup>

$$\pi = -\frac{RT}{\omega_o} \left[ \ln(1 - \theta_1 - \theta_2) + \theta_1 \left( 1 - \frac{1}{n_1} \right) + \theta_2 \left( 1 - \frac{1}{n_2} \right) + a_1 \theta_1^2 + a_2 \theta_2^2 + 2a_{12} \theta_1 \theta_2 \right] \quad (3.3)$$

$$b_1 c_1 = \frac{\theta_1}{(1 - \theta_1 - \theta_2)^{n_1}} \exp(-2a_1 \theta_1 - 2a_{12} \theta_2) \exp[(1 - n_1)(a_1 \theta_1^2 + a_2 \theta_2^2 + 2a_{12} \theta_1 \theta_2)] \quad (3.4)$$

$$b_2 c_2 = \frac{\theta_2}{(1 - \theta_1 - \theta_2)^{n_2}} \exp(-2a_2 \theta_1 - 2a_{12} \theta_1) \exp[(1 - n_2)(a_1 \theta_1^2 + a_2 \theta_2^2 + 2a_{12} \theta_1 \theta_2)] \quad (3.5)$$

Here  $\theta_i = \Gamma_i \omega_i$  is the monolayer coverage,  $\Gamma_i$  is the adsorption,  $\pi = \gamma_0 - \gamma$  is the surface pressure,  $\gamma_0$  is the surface tension of solvent,  $n_i = \omega_i / \omega_0$ ,  $\omega_i$  and  $\omega_0$  are the partial molar surface areas of the surfactant and solvent, respectively,  $a_1$ ,  $a_2$  and  $a_{12}$  are the constants of intermolecular interactions,  $b_i$  is the adsorption constant,  $c_i$  is the surfactant concentration in the solution bulk. Choosing the dividing surface after LucassenReynders (i.e. assuming  $\omega_0 = \omega$  and  $\Gamma_0 + \Gamma_1 + \Gamma_2 = 1/\omega$ , where  $\omega$  is the mean molar area of both surfactants, 1 and 2), one can eliminate the contributions which result from the entropic non-ideality of the solvent, thus

reducing Eq. (3) to a much simpler form<sup>[1]</sup>

$$\pi = -\frac{RT}{\omega}[\ln(1 - \theta_1 - \theta_2) + \alpha_1\theta_1^2 + a_2\theta_2^2 + 2a_{12}\theta_1\theta_2] \quad (3.6)$$

where

$$\omega = \frac{\omega_1\Gamma_1 + \omega_2\Gamma_2}{\Gamma_1 + \Gamma_2} \quad (3.7)$$

while the choice of the dividing surface after LucassenReynders does not affect the form of the adsorption isotherms, the values of  $n_i$  should now be calculated as  $n_i = \omega_i/\omega$ . To apply (3.4), (3.5), (3.6) and (3.7), one should possess information about the adsorption characteristics of the individual surfactants ( $b_i, a_i, \omega_i$ ). If no specific interaction exists between the molecules of different species, then one can approximately assume  $a_{12} = (a_1 + a_2)/2$ .

The values of  $b_i, \omega_i, a_i$  for the individual surfactant is calculated via best fit between experimental surface tension isotherm for individual solution, and the Frumkin model:<sup>[2,3]</sup>

$$\pi = -\frac{RT}{\omega_1}[\ln(1 - \theta_i) + a_i\theta_i^2] \quad (3.8)$$

$$b_i c_i = \frac{\theta_i}{(1 - \theta_i)} \exp(-2a_i\theta_i) \quad (3.9)$$

eqs 3.8 and 3.9 comes from the eqs 3.3 and 3.4 for  $\theta_2 = 0$ , provided that  $\omega_o = \omega_i$ .

For some systems the value of  $a_{12}$  bears an element of uncertainty as  $a_{12} \neq (a_1 + a_2)/2$  (*e.g.* mixture of  $C_{10}EO_5$  with SDS in 0.01 M NaCl, and mixture of betaine homologues BHB12+BHB16), and calculations using (3.4), (3.5), (3.6) and (3.7) cannot ensure a correct description of the experimental results for any mixture of surfactant. For that type of syemtems one much simpler theoretical models, which additionally account for processes actually taking place in the adsorption layer, can be used instead of the complicated set of (3.4), (3.5), (3.6) and (3.7) assuming that  $a_{12} = (a_1 + a_2)/2$ . Moreover, the use of this model does not require any preliminary consideration of isotherms of individual solutions,

but rather involves information about the surface tension at certain surfactant concentrations.

For that, an exact expression is being derived which relates the surface pressure of a surfactant mixture with the surface pressure of individual solutions. For an ideal ( $a_1 = a_2 = a_{12} = 0$ ) mixture of homologues ( $\omega = \omega_1 = \omega_2$ ), Eq. (3.6) can be rewritten in the following form:

$$\pi = \frac{RT}{\omega} \ln \left( \frac{\theta_1}{1 - \theta_1 - \theta_2} + \frac{\theta_2}{1 - \theta_1 - \theta_2} + 1 \right) = \frac{RT}{\omega} \ln(b_1 c_1 + b_2 c_2 + 1) \quad (3.10)$$

Using the corresponding equations for the individual surfactants 1 and 2 (in the following the subscript 0 refers to the individual solution),

$$\pi_i = -\frac{RT}{\omega} \ln(1 - \theta_{oi}) = \frac{RT}{\omega} \ln(1 + b_i c_i) \quad (3.11)$$

$$b_i c_i = \frac{\theta_{oi}}{(1 - \theta_{oi})} \quad (3.12)$$

and substituting the terms  $b_i c_i$  in Eq. (3.10) by the corresponding products from Eq. (3.11), one can express the equation of state [ Eq. (3.10)] as<sup>[1]</sup>

$$\exp \bar{\pi} = \exp \bar{\pi}_1 + \exp \bar{\pi}_2 - 1 \quad (3.13)$$

$\bar{\pi} = \pi\omega/RT$ ,  $\bar{\pi}_1 = \pi_1\omega/RT$ , and  $\bar{\pi}_2 = \pi_2\omega/RT$  are the dimensionless surface pressures of the mixture and individual solutions of components 1 and 2, respectively, at identical surfactant concentrations as in the mixture.

The use of Eq. (3.13) requires only the knowledge of one pair of experimental values of surface pressure (tension) of individual solutions. The only parameter of the isotherm in Eq. (3.13), the molar area  $\omega$ , can be either calculated from the molecular geometry of the surfactant, or determined experimentally from the limiting slope of the  $\gamma$  vs.  $\ln c$  curve for the individual surfactant (slope of the surface tension isotherm near the CMC or solubility limit). Moreover, it was shown that calculations according to Eq. (3.13) are quite insensitive with respect to the choice of  $\omega$ . Therefore for estimation one can assume that this parameter is equal to the value typical for the surfactant type considered, e.g.  $(1.2 \div 2.0) \times 10^5 \text{ m}^2/\text{mol}$  for fatty alcohols, acids, ethers and most anionic and cationic surfactants.

## 3.2 Simple Model

### 3.2.1 Non Ideal Mixture of homologues

The equation of state and adsorption isotherm for a non-ideal surface layer of two surfactants with the same molar areas are Eq. (3.6) and

$$b_i c_i = \frac{\theta_i}{(1 - \theta_1 - \theta_2)} \exp(-2a_i \theta_i - 2a_{12} \theta_i) \quad (3.14)$$

It can be shown that Eq. (3.13) can be used also for the approximate description of non-ideal mixtures of surfactants. The  $\pi$  value in Eq. (3.6) are being expressed via the terms  $b_i c_i$  using the expressions for isotherms Eq. 3.14. These products are being expressed via the  $\pi_i$  values for individual solutions using (3.8) and (3.9). This leads to a (dimensionless) equation of state for a mixed surface layer:

$$\exp \bar{\pi} = k_1 \exp \bar{\pi}_1 + k_2 \exp \bar{\pi}_2 - k_3 \quad (3.15)$$

here

$$\begin{aligned} k_1 &= \exp[a_1(\theta_{o1}^2 - \theta_1^2 + 2\theta_1 - 2\theta_{o1}) - a_2\theta_2^2], \\ k_2 &= \exp[a_2(\theta_{o2}^2 - \theta_2^2 + 2\theta_2 - 2\theta_{o2}) - a_1\theta_1^2] \text{ and} \\ k_3 &= \exp(2a_1\theta_1 - a_1\theta_1^2 - a_2\theta_2^2) + \exp(2a_2\theta_2 - a_1\theta_1^2 - a_2\theta_2^2) - \exp(-a_1\theta_1^2 - a_2\theta_2^2), \end{aligned}$$

the subscript 0 refers to the coverage for the individual solution. Eq. (3.15) and the expressions for  $k_i$  is obtained by (3.6) and (3.14) for  $a_{12} = 0$ . If the more correct expression  $a_{12} = (a_1 + a_2)/2$  is introduced, then the resulting equation converts into the Eq. (3.15), but the expressions for  $k_i$  become more cumbersome. Analysing the values of  $k_i$  one can see that, as the signs of the terms in the expression are opposite, and assuming only small deviations from ideality, one obtains the approximation  $k_1 \cong k_2 \cong k_3 \cong 1$ . If the condition  $a_{12} = (a_1 + a_2)/2$  is imposed, all coefficients  $k_i$  are also become approximately equal to 1. Therefore Eq. (3.15) which describes a non-ideal surface layer can be approximately transformed into Eq. (3.13) derived for an ideal mixture of homologues. For ideal surface layers, as  $a_1 = a_2 = 0$  and thus  $k_1 = k_2 = k_3 = 1$ , Eq. (3.15) coincides exactly with Eq. (3.13).<sup>[1]</sup>

### 3.2.2 Mixtures of ionic homologues

Mixtures of an ionic and a non-ionic surfactant are quite similar to those considered above, because the contribution of the DEL to the surface pressure for an ionic surfactant can be treated approximately as a non-ideality factor.<sup>[1]</sup> If the solution of an individual ionic surfactant and their mixtures are studied at fixed ionic strength (which is usually accomplished by addition of an indifferent electrolyte), then the surface pressure of a mixed solution, similarly to non-ionic surfactants, is calculated from the data obtained for individual solutions according to Eq. (3.13)

It is known that mixtures of ionic surfactants for any arbitrary ionic strength can be satisfactorily described by the Langmuir or Frumkin equation of state and isotherm if, instead of the concentrations of ionic surfactants, the electroneutral combination of the corresponding ions is considered.

Considering a solution containing a mixture of two anionic (or cationic) surfactants, e.g. the two homologues  $R_1X$  and  $R_2X$  with a common counterion  $X^+$ , with or without addition of an electrolyte  $XY$ . In such systems the counterion concentration of  $X^+$  is given by the sum of the concentrations of  $R_1X$ ,  $R_2X$  and  $XY$ . For an ideal surface layer ( $a_1 = a_2 = a_{12} = 0$ ) the following surface pressure isotherm is obtained:

$$\pi = \frac{RT}{\omega_{RX}} \ln[(b_1 f_{1\pm})^2 c_{R_1X} c_{X^-} + (b_2 f_{2\pm})^2 c_{R_2X} c_{X^-}]^{1/2} + 1] \quad (3.16)$$

$$\pi_i = \frac{RT}{\omega_{R_iX}} \ln[(b_i f_{i\pm})^2 c_{R_iX} c_{X^-}]^{1/2} + 1] \quad (3.17)$$

Combining (3.16) and (3.17), one obtains the expression which describes the surface tension of a mixed solution

$$\exp \overline{\pi} = ((\exp \overline{\pi}_1 - 1)^2 + (\exp \overline{\pi}_2 - 1)^2)^{1/2} + 1 \quad (3.18)$$

Unlike the cases of non-ionic surfactants or ionic surfactants in the presence of electrolyte excess, when the calculations according to Eq. (3.13) are made, one should choose the concentrations (activities) of the individual solutions to ensure that the products of the concentrations of the corresponding surface-active

ions and counterions in the individual solutions and in the mixture of surfactants are equal. For example, assuming that the  $R_1X$  concentration in the mixture is  $c_1$ , and the concentration of  $R_2X$  is  $c_2$ , then the surface pressure in the individual solutions is determined for the concentrations  $c_{01} = [c_1(c_1 + c_2)]^{1/2}$  and  $c_{02} = [c_2(c_1 + c_2)]^{1/2}$ , respectively. This means, the increase of the counterion concentrations in the surfactant mixture due to the addition of the second surfactant's counterion has to be taken into account. Similarly the influence of small additions of indifferent electrolyte with a common counterion has also to be considered.<sup>[1]</sup>

The account for non-ideality of the components in the surface layer of an ionic mixture does not affect the form of Eq. (3.18). Thus, Eq. (3.13) in the case of counterion excess, and Eq. (3.18) in the case where no indifferent electrolyte is added (and the correction of the concentrations in the individual solutions as described above) are allowed an approximate description of non-ideal mixtures of ionic surfactants with identical values of the molar areas of the components.

### 3.2.3 Mixture of components with different molar areas

The surfactants with essentially different molar areas are often present simultaneously in a mixture. The equation of state for the ideal mixture of two surfactants with different  $\omega_i$  is also expressed by Eq. (3.10) in the form

$$\pi = -\frac{RT}{\omega} \ln(1 - \theta_1 - \theta_2) \quad (3.19)$$

However, the  $\omega$  value is no longer a constant but depends on the corresponding partial molar areas  $\omega_i$  and adsorptions of components 1 and 2, and is given by Eq. (3.7). For this mixture, the adsorption isotherms are

$$b_i c_i = \frac{\theta_i}{(1 - \theta_1 - \theta_2)^{n_i}} \quad (3.20)$$

where  $n_i = \omega_i/\omega$ . Following the lines of the preceding sections, one can use Eq. (3.20) to substitute the  $b_i c_i$  terms in Eq. (3.19), and then by the corresponding values of  $b_i c_i$  for individual solutions from Eq. (3.11). This again results in Eq. (3.15) where the coefficients are being defined by

$$k_1 = (1 - \theta_1 - \theta_2)^{1-n_1},$$

$$k_2 = (1 - \theta_1 - \theta_2)^{1-n_2} \text{ and}$$

$$k_1 = (1 - \theta_1 - \theta_2)^{1-n_1} - (1 - \theta_1 - \theta_2)^{1-n_2}.$$

If the values  $n_i$  are close to 1, then the coefficients  $k_i$  can be approximated to be 1, i.e. Eq. (3.13) is valid also for this case. The dimensionless pressures in Eq. (3.13) are  $\bar{\pi} = \pi\omega/RT$ ,  $\bar{\pi}_1 = \pi_1\omega/RT$  and  $\bar{\pi}_2 = \pi_2\omega/RT$ . To determine the average of  $\omega$  for the mixture, the same substitution procedure for  $b_i c_i$  is used and Eq. (3.7) becomes

$$\omega = \frac{\omega_1(\exp\bar{\pi}_1 - 1) + \omega_2(\exp\bar{\pi}_2 - 1)(1 - \theta_1 - \theta_2)^{n_2-n_1}}{\exp\bar{\pi}_1 - 1 + \exp\bar{\pi}_2 - 1)(1 - \theta_1 - \theta_2)^{n_2-n_1}} \quad (3.21)$$

Estimations using this equation show that even if the difference between the  $\omega_i$  values is relatively small, but the surface coverage is medium or large, the term  $(1 - \theta_1 - \theta_2)^{n_2-n_1}$  cannot be omitted. However, one can use other expressions for the mean area, which do not involve unknown quantities. As the adsorption is roughly proportional to the surface pressure (for extremely diluted surface layers  $\pi = RT\Gamma$ ), then the mean molar area, Eq. (3.7), is given by

$$\omega = \frac{\omega_1\pi_1 + \omega_2\pi_2}{\pi_1 + \pi_2} = \omega_1 \frac{\bar{\pi}_1 + \bar{\pi}_2}{\bar{\pi}_1 + \bar{\pi}_2(\omega_1/\omega_2)} \quad (3.22)$$

Assuming that the adsorptions of surfactants characterized by different molar areas in a densely packed layer (at sufficiently large  $\pi$ ) are roughly inverse proportional to  $\omega_i$ , instead of Eq. (3.22) one obtain

$$\omega = \frac{\omega_1\pi_1/\omega_1 + \omega_2\pi_2/\omega_2}{\pi_1/\omega_1 + \pi_2/\omega_2} = \omega_1 \frac{\pi_1 + \pi_2}{\pi_1 + \pi_2(\omega_1/\omega_2)} \quad (3.23)$$

Both models provide quite a satisfactory description of surfactant mixtures with essentially (more than twice) different values of the molar areas of the components.

### 3.3 Other Model

The Joos relationship for the equation of state to describe the mixed adsorption behaviour is given by following equation (This relation for surfactants in concentration up to about  $10^{-2} \text{ mol/liter}$ )<sup>[4]</sup>

$$\exp\left(\frac{-\pi}{RT\Gamma_0^\infty}\right) + \exp\left(\frac{-\pi}{RT\Gamma_1^\infty}\right) \frac{C_1}{a_1} + \exp\left(\frac{-\pi}{RT\Gamma_2^\infty}\right) \frac{C_2}{a_2} = 1 \quad (3.24)$$

where  $\Gamma_0^\infty$ ,  $\Gamma_1^\infty$  and  $\Gamma_2^\infty$  are the maximum adsorption of the solvent, and surfactant 1 and 2, respectively,  $\pi$  is the surface pressure,  $R$  is the gas law constant and  $T$  is the absolute temperature. The parameters  $a_1$  and  $a_2$  are expressed as

$$a_1 = \exp\left(\frac{\mu_1^S - \mu_1^B}{RT}\right) \omega \quad (3.25)$$

$$a_2 = \exp\left(\frac{\mu_2^S - \mu_2^B}{RT}\right) \omega \quad (3.26)$$

where  $\mu^S$  is the chemical potential in the surface under standard conditions,  $\mu^B$  is the chemical potential in the bulk under standard conditions and  $\omega$  is the number of the moles of water per litre. Eq/ 3.24 gives good results for the systems of nonionic or ionic surfactants in the presence of a swamping electrolyte.

If one of the surfactant is a 1:1 electrolyte type  $A^+$  and  $B^-$ , then its activity is close to  $10^{-2}M$ . Therefore for an aqueous solution including one nonionic and one ionic surfactant type 1:1 electrolyte in the absence of a swamping electrolyte, eq 3.24 is approximately written in the form

$$\exp\left(\frac{-\pi}{RT\Gamma_0^\infty}\right) + \exp\left(\frac{-\pi}{RT\Gamma_1^\infty}\right) \frac{C_1}{a_1} + \exp\left(\frac{-\pi}{2RT\Gamma_2^\infty}\right) \frac{C_2}{a_2^*} = 1 \quad (3.27)$$

where

$$a_2^* = \exp\left(\frac{\mu_2^S - \mu_2^B}{2RT}\right) \omega \quad (3.28)$$

assuming that  $C_2C_1 = b = \text{const}$  and  $C_{tot} = C_1 + C_2 = C_1(1 + b)$  gives

$$\exp\left(\frac{-\pi}{RT\Gamma_0^\infty}\right) + \left[\exp\left(\frac{-\pi}{RT\Gamma_1^\infty}\right) \frac{1}{a_1} + \exp\left(\frac{-\pi}{2RT\Gamma_2^\infty}\right) \frac{b}{a_2^*}\right] \frac{C_{tot}}{1 + b} = 1 \quad (3.29)$$

The values of  $\Gamma_0^\infty$ ,  $\Gamma_1^\infty$ ,  $\Gamma_2^\infty$ ,  $a_1$  and  $a_2^*$  used in eq 3.29 are calculate from the data for the individual surfactants on the assumption that  $C_1 = 0$  or  $C_2 = 0$ . So by using eq 3.29 one can calculate the value of surface pressure.

# Chapter 4

## Results

In the previous chapter it was seen that suitable models were taken from the literature reports, thereby making a precise data analysis in an attempt to compare the experimental results with those coming out from the literature survey.

### 4.1 CMC prediction

For CMC prediction the following model was used:

$$CMC = x_1^m f_1 CMC_1 + (1 - x_1^m) f_2 CMC_2 \quad (4.1)$$

where  $f_1$  and  $f_2$  are the activity coefficients of the surfactants in the micelle. For calculation of these activity coefficients following equation are used<sup>[5]</sup>

$$\ln f_1 = (1 - x_1^m)^2 \beta \quad (4.2)$$

$$\ln f_2 = (x_1^m)^2 \beta \quad (4.3)$$

in terms of solution compositions

$$\frac{1}{CMC} = \frac{x_1}{f_1 CMC_1} + \frac{1 - x_1}{f_2 CMC_2} \quad (4.4)$$

and

$$x_1^m = \frac{x_1 f_1 CMC_2}{x_1 f_2 CMC_2 + (1 - x_1) f_1 CMC_1} \quad (4.5)$$

Results are attached

## 4.2 Surface Tension Prediction

For surface tension prediction following model was used

$$\pi = -\frac{RT}{\omega} [\ln(1 - \theta_1 - \theta_2) + \alpha_1 \theta_1^2 + a_2 \theta_2^2 + 2a_{12} \theta_1 \theta_2] \quad (4.6)$$

where

$$\omega = \frac{\omega_1 \Gamma_1 + \omega_2 \Gamma_1}{\Gamma_1 + \Gamma_2} \quad (4.7)$$

Here  $a_{12} = \frac{a_1 + a_2}{2}$  if this is not valid then following eqs is used<sup>[1]</sup>

$$\exp \bar{\pi} = \exp \bar{\pi}_1 + \exp \bar{\pi}_2 - 1 \quad (4.8)$$

For individual solutions surface tension isotherm Frumkin model is used

$$\pi = -\frac{RT}{\omega_1} [\ln(1 - \theta_i) + a_i \theta_i^2] \quad (4.9)$$

$$b_i c_i = \frac{\theta_i}{(1 - \theta_i)} \exp(-2a_i \theta_i) \quad (4.10)$$

Results are attached

# Chapter 5

## Conclusions

We have used 2 models for the CMC prediction and Surface tension prediction.

Model used for CMC prediction is the pseudophase separation model. In this approach the micelles are treated as a separate, infinite phase in the equilibrium with the monomer phase. We have predicted the values of CMC for the *SDS* – *C<sub>8</sub>E<sub>4</sub>*, SDS-DeTAB and *C<sub>12</sub>SO<sub>4</sub>Na* – *C<sub>12</sub>E<sub>8</sub>* system. Results are good agreement with experimental data reported in the literature.

We have predicted the Surface tension values for the 1-heptanol-1-octanol, SDS-1-butanol, SDS-1-nonanol systems. and these values match from the experimental values.

So we can conclude that one can predict the values of Surface tension and CMC for a surfactant mixture by using various models available in literature.

# References

1. Fainerman, V. B., Miller R., and Aksenenko E. V., 2002, "Simple model for prediction of surface tension of mixed surfactant solutions," *Advance in Colloid and Interface Science* , **96**, 339-359
2. Fainerman, V. B., Miller R., and Aksenenko E. V., 2000, "Adsorption Behaviour of oxyethylated alcohols at the solution/air interface ," *Langmuir* , **16**, 4196-4201
3. Fainerman, V. B., Miller R., and Aksenenko E. V., 2002, "Effect of aggregation in the adsorption layer at the liquid /fluid interface on the shape of the surface pressure isotherm," *J. Phys. Chem.*, **104**, 5744-5749
4. Janczuk, B., Bruque J. M., Gonzalez-Martin M. L., and Dorado-Calasanz C., 1995, "The properties of mixtures of ionic and nonionic surfactants in water at the water/air interface." *Colloids and Surface*, **104**, 157-163
5. Jonsson, Lindman, Holmberg, and Kronberg, 1988, " Surfactants and Polymers in Aqueous solution," *John Wiley and sons* , 115-133
6. Holland Paul M., 1986, "Nonideal Mixed micellar solutions," *Advances in colloid and Interface Science.*, **26**, 111-129
7. Holland Paul M., 1992, "Mixed surfactant systems," *ACS Symposium Series.* **501**, 31
8. Shiloach, Anat., and Blankshtein. Daniel., 1998, " Predictiong micellar solution properties of binary surfactant mixtures, " *Langmuir.* , **14**, 1618-1636

9. Sudhakar Puvvada., and Blankschtein Daniel.,1992 “Thermodynamic description of Micellization, phase behaviour, and phase separation of aqueous solutions of surfactant mixtures,” *J. Phys. Chem.* **96** 5567-5579
10. Sudhakar Puvvada., and Blankschtein Daniel., 1992 “Theoretical and experimental investigations of micellar properties of aqueous solutions containing binary mixtures of nonionic surfactants,” *J. Phys. Chem.* **96** 5567-5579